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ON THE SEPARATION OF BOUNDARY LAYERS

By V. A. Sandborn

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

SUMMARY

By using an assumed empirical form for the mean velocity distribution as a guide, a model for turbulent separation is postulated. This postulated model, which requires that the outer region of the turbulent boundary layer has completely taken over the entire layer at separation, leads to a direct connection between the turbulent mean velocity profile and an equivalent laminar boundary-layer profile.

The analysis yields a separation criterion that relates the form factor at separation to the ratio of the displacement to the boundary-layer thickness. This criterion reduces to a variation of form factor with the Pohlhausen pressure-gradient parameter for laminar flow. The criterion for laminar separation is also an upper limit for intermittent turbulent separation.

SYMBOLS

- A, B, C constants
 C_f skin friction coefficient, for flat plate, $\tau_w / \frac{1}{2} \rho U_1^2$
 c chord of airfoil
 $f(H)$ function of H defined by equation (15)
 H profile form factor, δ^*/θ
 L length of flat plate

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M	Mach number
m	profile parameter depending on outer region of boundary layer
n	profile parameter depending on inner region of boundary layer
P	static pressure
q_1	dynamic pressure
S_1	distance along upper surface of airfoil, measured from the leading edge
s	wall shear stress parameter, $\delta\tau_w/2U_1\mu$
U	local mean velocity
U_1	free-stream mean velocity
X	distance along curved surface of elliptic cylinder
x	direction parallel to boundary and in direction of mean flow
y	direction normal to boundary and approximately normal to mean flow
α	angle of attack
δ^*	boundary-layer displacement thickness
δ	boundary-layer thickness
ζ	profile parameter, defined in eq. (3)
θ	boundary-layer momentum thickness
λ	Pohlhausen pressure-gradient parameter, $(\delta^2/\nu) dU_1/dx$
μ	coefficient of viscosity
ν	kinematic viscosity, μ/ρ
ρ	density of air
τ_w	wall shear stress

Subscripts:

- 1 ahead of normal shock
- 2 behind normal shock

INTRODUCTION

Prediction of turbulent boundary-layer separation is still a fundamental problem of fluid mechanics. While empirical methods (notably the method devised by Maskell (ref. 1) and improved upon by several recent authors) exist for predicting separation, little fundamental understanding of the parameters governing separation is gained from the predictions. Only for abrupt separations due to shock waves in supersonic flow has any systematic study of separation been made (ref. 2).

The present paper is concerned with the relation between the various parameters representing a boundary layer at separation. While empirical in nature, the analysis leads to an accurate criterion for separation in terms of the profile parameters. Laminar, as well as turbulent separation is included in the analysis. The results will be shown to be quite useful in analyzing experimental data, and are presented herein with the hope that further use may be found for the criterion in predicting separation.

ANALYSIS

In order to specify the profile parameters at separation, a general form for the mean velocity distribution is first assumed. At separation certain simplifications are made, which lead to relations between the

parameters. The analysis proceeds apart from the equations of motion, thus the results depend heavily upon the flow model proposed. The model in turn may be judged by how well the final results can represent experimental data.

Velocity Profile Equation

A general relation for the mean velocity profile was studied in reference 3 and is assumed for the starting point of the present analysis. This relation is

$$U/U_1 = A + B(1 - y/\delta)^m + C(1 - y/\delta)^{2n} \quad (1)$$

The form of the equation is that suggested by Pai (ref. 4) to represent the velocity distribution in pipes and channels. The modification of equation (1) for the boundary layer was outlined in (ref. 3).

The constants A, B, and C of equation (1) are evaluated from the following boundary conditions, which are valid for both turbulent and laminar boundary layers:

$$y = 0 \quad \left\{ \begin{array}{l} U = 0 \\ \partial U / \partial y = \tau_w / \mu \end{array} \right. \quad (2)$$

at

$$y = \delta \quad \left\{ \begin{array}{l} U = U_1 \\ \partial U / \partial y = 0 \end{array} \right.$$

The values of A, B, and C are

$$\left. \begin{aligned} A &= 1 \\ B &= \frac{2(s - n)}{2n - m} \equiv \zeta \\ C &= -\frac{2s - m}{2n - m} = -(1 + \zeta) \end{aligned} \right\} \quad (3)$$

where s is a fundamental wall shear stress parameter defined as, $s = \delta \tau_w / 2U_1 \mu$. The parameter ζ , as noted in equation (3) is used for the value of the particular grouping of s , m , and $2n$ encountered. Equation (1) is rewritten as

$$U/U_1 = 1 + \zeta(1 - y/\delta)^m - (1 + \zeta)(1 - y/\delta)^{2n} \quad (4)$$

The constants ζ , m , and $2n$ are related to the familiar boundary-layer parameters, displacement and momentum thickness, by the following equations:

$$\delta^*/\delta = -\frac{\zeta}{m+1} + \frac{1+\zeta}{2n+1} \quad (5)$$

and

$$\frac{\theta}{\delta} = -\frac{\zeta}{m+1} - \frac{\zeta^2}{2m+1} + (1 + \zeta) \left[\frac{2\zeta}{2n+m+1} + \frac{1}{2n+1} - \frac{(1+\zeta)}{4n+1} \right] \quad (6)$$

The general characteristics of the profile equation were demonstrated in reference 3, and are briefly outlined here.

Equation (4) is the general form of the assumed velocity profile. As it stands, it requires a knowledge of δ , τ_w , U_1 , μ , m , and $2n$. In order that the first velocity derivatives be finite, as specified by the boundary conditions (2), m and $2n$ must be positive numbers greater than one. Higher order velocity derivatives may of course not be reasonable from the present relation, if either m or $2n$ is less than 2.

Turbulent Flow

For the case of turbulent flow, equation (4) represents a profile in which two regions of importance are defined.

The outer major portion of the profile. - This region is almost completely specified by the m power term. This is accomplished mathematically by m being of order unity, while $2n$ is at least an order of magnitude greater.

Under certain flow conditions the outer region of measured turbulent boundary-layer mean velocity profile was found by Clauser (refs. 5 and 6) to demonstrate similarity. For the conditions needed to produce similarity profiles, termed equilibrium flows by Clauser, it was demonstrated in reference 3 that m was a constant for each equilibrium flow and related directly to the constants Clauser used to describe the flow. Further, Clauser (ref. 6) demonstrated that the similar profiles could be related to equivalent laminar profiles. Following Clauser's results, it was demonstrated in reference 3 that m could be obtained directly from the known equivalent laminar solution.

The inner region very near the wall. - In this region both m and $2n$ power terms are of importance. For this inner region of the turbulent boundary layer the present relation was found to agree accurately with the experimental measurements. The measurements in turn may in some cases agree with the universal logarithmic law of the wall, and in other cases deviate considerably from the logarithmic distribution. The present velocity function, in close agreement with the measurements, questions the validity of the logarithmic distribution.

Laminar Flow

For laminar flow it was possible to simplify the general relation (4). Two separate power terms are not needed in laminar flow. Thus, the function at the indeterminate point $2n = m$ proved to be an accurate representation of laminar profiles. The limit of equation (4) as $2n \rightarrow m$ yields the following relation (s and δ held constant):

$$U/U_1 = 1 + (1 - y/\delta)^m [(2s - m) \log (1 - y/\delta) - 1] \quad (7)$$

A special case, $2s = m$, reduces to the known exact solutions for laminar flow.

$$U/U_1 = 1 - (1 - y/\delta)^m \quad (8)$$

- (a) $m = 0$, potential flow
- (b) $m = 1$, laminar Couette flow
- (c) $m = 2$, laminar Poiseuille flow

(Note that eq. (8) would also be obtained directly from eq. (4) by setting $2s = m$.)

Equation (7) may be expressed in terms of the Pohlhausen pressure gradient parameter λ by requiring that

$$\mu \left[\frac{\partial^2 U}{\partial y^2} \right]_{y=0} = \frac{\partial P}{\partial x}$$

The resulting two-parameter family for laminar profiles is

$$U/U_1 = 1 + (1 - y/\delta)^m \left[\frac{m(m-1)}{2m-1} \frac{-\lambda}{\log(1 - y/\delta) - 1} \right] \quad (9)$$

When $\delta = 5.2 \sqrt{\nu x/U_1}$, and $\lambda = 0$, and when $m (= 2.86)$ is evaluated from the momentum equation, a comparison of equation (9) with the Blasius results is given in the following table:

Present relation		Exact values (Blasius)
C_f	$1.33 \sqrt{\nu/U_1 L}$	$1.328 \sqrt{\nu/U_1 L}$
θ	$0.664 \sqrt{\nu x/U_1}$	$0.664 \sqrt{\nu x/U_1}$
δ^*	$1.74 \sqrt{\nu x/U_1}$	$1.729 \sqrt{\nu x/U_1}$

Laminar Separation

The problem of laminar separation can be attacked by theoretical methods with reasonable success. Howarth, (ref. 7) and Hartree (ref. 8) have obtained solutions for particular pressure gradient flow, which yield separation profiles. Gortler (ref. 9) has recently developed a series solution for the laminar-boundary-layer equations of motion, which is valid for arbitrary pressure gradients. The solution of Gortler should allow a general type of boundary layer development and separation to be predicted. There is, of course, a possibility that the boundary-layer equations are not valid in the separation region, and as such the theoretical predictions are questionable. Certainly, the following analysis for laminar flow is regarded as a check of the assumed velocity-profile function, and is not suggested as an improvement of the existing methods of separation prediction.

The laminar separation profile may be obtained directly from equation (7) by requiring the skin friction to be zero (or $\partial U/\partial y \Big|_{y=0} = 0$). This boundary condition leads to

$$m = \pm \sqrt{-\lambda} \quad (10)$$

where in application only the plus sign has physical meaning. The velocity-profile equation for laminar separation becomes

$$U/U_1 = 1 + (1 - y/\delta) \sqrt{-\lambda} \left[\sqrt{-\lambda} \log (1 - y/\delta) - 1 \right] \quad (11)$$

The relation for displacement and momentum thickness becomes

$$\delta^*/\delta = \frac{(2\sqrt{-\lambda} + 1)}{(\sqrt{-\lambda} + 1)^2} \quad (12)$$

and

$$\frac{\theta}{\delta} = \frac{(2\sqrt{-\lambda} + 1)}{(\sqrt{-\lambda} + 1)^2} - \frac{2(\sqrt{-\lambda})^2}{(2\sqrt{-\lambda} + 1)^3} - \frac{2\sqrt{-\lambda}}{(2\sqrt{-\lambda} + 1)^2} - \frac{1}{(2\sqrt{-\lambda} + 1)} \quad (13)$$

These laminar separation equations do not require a particular value of λ at separation. This is in keeping with the experimental facts that separation is found to occur over a range of values of λ between approximately -5 and -12.

The profiles predicted by equation (11) are compared with several well-known laminar separation profiles in figure 1. The value of λ in each case was taken from information known concerning the profile being compared (for the theoretical profiles δ , given on fig. 1, is at $U/U_1 = 0.995 \pm 0.001$). In figures 1(a) and (b) the theoretical profiles computed by Howarth (ref. 7), and Hartree (ref. 8) for particular pressure gradients are compared with the present relation. Figure 1(c) is a comparison with the Pohlhausen separation profile for $\lambda = -12$. Figure 1(d) compares the present relation with actual measurements made by Schubauer (ref. 10) in the separation region of an elliptic cylinder.

The agreement of the present profile with the two theoretical profiles is not good. There is the possibility, as pointed out by Howarth (ref. 7), that the basic boundary-layer assumption of conditions changing very slowly in the x-direction is not valid, so that the theoretical profiles are questionable. The comparison of the present profile with the measurements of Schubauer (fig. 1(d)) is not exact; however, a value of $\lambda = -6$ instead of the -5.03 originally given by Schubauer would result in very close agreement. The value of λ reported by Schubauer has been questioned by several investigators, since it did not lead to a prediction of separation (e.g., ref. 11).

As a criterion for laminar separation, equations (12) and (13) may be combined to give an expression for the boundary-layer form factor, $H = \delta^*/\theta$, as a function of $\sqrt{-\lambda}$. This relation is plotted in figure 2. For the particular profiles just considered in figure 1, the criterion is quite reasonable. The asymptotic value of H for an infinite pressure gradient is 2.667. This limiting value of H is slightly higher than that for the best known nonseparating profile, the Blasius boundary layer, in which $H = 2.60$. Included in figure 2 are the data from several turbulent separation profiles, which will be discussed later.

Turbulent separation. - The understanding of turbulent separation is quite meager. Indeed there is still a problem of how separation is to be defined for analytical studies. Kline (ref. 12) in recent years has shown qualitatively from water channel observations that at least two types of turbulent separation may be found. By defining separation

as the point where "any" backflow of fluid occurs near the wall, Kline observed that separation first begins in a very unsteady way. This first type of separation, which will be called intermittent separation, will have a velocity gradient other than zero at the wall on a mean basis and therefore will have a value of skin friction other than zero. Thus, intermittent separation can not be predicted by assuming zero skin friction. The second type of separation occurs downstream of the first, and corresponds closely to the classical concept of steady separation with zero skin friction.

An example of the effect of intermittent separation is indicated by the measurements shown in figure 3. While it was originally assumed that the failure to measure a zero skin friction was a fault of the measuring technique, it is now apparent that the nonzero skin friction is due in part to the separation being intermittent. Maskell (ref. 1) has suggested use of the empirical Ludwig-Tillmann skin friction equation, (ref. 14) to obtain a curve such as that in figure 3, and then extrapolated the curve directly to zero skin friction (since the Ludwig-Tillmann equation does not give a zero value for skin friction) to indicate the point of separation. Thus, Maskell has determined a point near the start of intermittent separation rather than the actual point of zero skin friction. Obviously, the start of intermittent separation is the practical point on which to base engineering predictions.

The present general profile relation (eq. (4)) appears capable of representing profiles in either the intermittent or two-dimensional

separation region (ref. 3). However, only for the intermittent separation region can a substantial simplification be made to the complex general relations of equations (4), (5), and (6). The simplification, which is based on hindsight obtained by actually trying to fit the general profile equation to experimental separation profiles, requires some assumptions as to the flow model in the intermittent separation region.

The general flow model described by equation (4) is a boundary layer with two regions of importance. The two regions are quite different in magnitude, with the outer region being the dominant part of the layer. It is assumed that at intermittent separation the dominance of the outer region of the layer has increased to such a magnitude that the inner sublayer can be considered to have vanished completely. The effective vanishing of the sublayer is in keeping with the concept that its non-dimensional thickness decreases as the x-Reynolds number increases. From equation (4) it may be seen that the inner sublayer can be neglected if the $2n$ power term is negligible. Mathematically this is accomplished by requiring $2n \rightarrow \infty$. The simple profile results.

$$U/U_1 = 1 - (1 - y/\delta)^m \quad (14)$$

By evaluating the velocity derivative at the wall in equation (14) the result $2s = m$ is obtained.

At this point a clear picture of turbulent separation can be postulated. The basic assumption is that the outer region has completely taken over the entire layer at separation. This may be an over simplification of the actual flow, but it will be shown to give accurate

results. The conditions existing in the outer region of the turbulent boundary layer have been studied extensively by Clauser (ref. 6). In particular, Clauser has demonstrated that this outer region is related directly to an equivalent laminar layer. Thus, the outer region of the turbulent boundary layer can be treated as a laminar layer with a very large value of viscosity. In what follows it is assumed that the equivalence is valid for any turbulent boundary layer. This assumption then requires that the turbulent separation profile, in which the inner layer has vanished, is identical to some laminar boundary layer. Since the turbulent separation is intermittent, presumably partly because the turbulent fluctuations of the outer region reach the wall, the corresponding laminar profile must still have a shear stress at the wall. The laminar profile corresponding to the turbulent intermittent separation profile would be expected to fall somewhere below the laminar separation criterion shown in figure 2. This turbulent separation model then requires that the laminar separation criterion is also an upper limit criterion for intermittent turbulent separation. The turbulence data shown in figure 2 seems to agree reasonably well with such a criterion. The discrepancy at large $\sqrt{-\lambda}$ may illustrate the degree of accuracy of the present empirical velocity profile relation.

For the present model the solution for turbulent separation is reduced to finding the equivalent laminar profile. Equation (14) may be thought of as a possible first approximation for the solution of the laminar boundary layer because as noted previously, equation (8) or equation (14) is the exact solution for the special cases of simple

laminar flows. This may explain why the foregoing analysis indicates that the simple relation, equation (14), represents the turbulent separation distribution.

A comparison of equation (14) with the turbulent separation profile measured by Schubauer and Klebanoff (ref. 15) is shown in figure 4.

Equation (14) does not represent the measurements in minute detail. However, it yields an accurate relation between the integral profile parameters. By evaluating the momentum and displacement thickness from equation (14) the following relation is obtained:

$$H = \delta^*/\theta = 1 + \frac{1}{1 - \delta^*/\delta} \quad (15)$$

Since equation (14) was suggested as the turbulent velocity profile at intermittent separation, it follows that equation (15) must be the relation between form factor H and δ^*/δ at separation. A plot of equation (15) together with the equivalent laminar separation criterion curve obtained from the laminar profile parameters (eqs. (12) and (13)) is shown on figure 5. Data from several measured separation profiles are compared with the separation criterions. These measurements are from a wide range of different flow configurations. In all cases the predicted criterion is reasonable and well within the limits of experimental uncertainty, thus it appears the assumed flow model is justified.

APPLICATION

Criterion (fig. 5) for both laminar and turbulent separation have been demonstrated. In neither case was the analysis carried far enough

to allow direct engineering predictions to be made from free-stream conditions only. There are, however, certain areas where the present results are of direct interest.

Prediction of Separation From Experimental Surveys

The separation criterion is of value in analyzing experimental data, since it can be used to quickly determine the location of a profile with respect to the start of separation. An extensive set of data was reported by Robertson and Holl (ref. 16) for turbulent boundary-layer flow in an axisymmetric diffuser. For this data of Robertson and Holl no statement about separation was made; however, if the values of skin friction coefficient listed in table II of reference 16 are plotted against x , it is noted that a definite change of slope occurs in the curves near $C_f = 0.0004$. Identifying the points above and below $C_f = 0.0004$ as tailed or untailed, respectively, it is shown in figure 6 that the change of slope was associated with the start of intermittent separation.

A second instance where the present results were employed was for the reported laminar separation bubbles on the nose of airfoils. Here certain unexpected results were found, which may cast doubt on the interpretation of this type of separation. Figure 7 shows a plot of the laminar and turbulent separation criterions compared with the experimental measurements at the start of "laminar" separation bubbles reported by Gault (ref. 17) and Bursnall and Loftin (ref. 18). The data for the NACA 66₃-018 airfoil at an angle of attack of 2° (the circle symbols on fig. 7) show that the value of H increased from an unseparated turbulent

value to just greater than the turbulent separation value. Also of interest is the case of the NACA 66₃-018 airfoil at zero angle of attack and a Reynolds number of 2.4×10^6 , reported by Bursnall and Loftin (the right triangle symbols on fig. 7). As can be seen in figure 7 the first such data point is very near the laminar separation curve and the second point, which is just downstream, jumps to the turbulent separation curve. The present analysis would suggest that most of these separation profiles are in the transition region between laminar and turbulent flow. In particular, it is possible that the data reported by Gault (circle and square symbols) (in most cases two points are included for each flow condition to indicate the variation of the parameters in this separation region), are actually turbulent separation profiles.

Figure 8(a) compares Gault's measured "laminar separation" profile for the NACA 66₃-018 airfoil at an angle of attack of 2° , a Reynolds number of 2×10^6 and $S_1/c = 0.612$ with both the laminar and turbulent separation profiles. The value of δ^*/δ obtained from the measured profile was used to evaluate the empirical curves. Likewise, figure 8(b) compares the "laminar separation" profile measured by Bursnall and Loftin for the same airfoil at zero angle of attack, $x/c = 0.61$, and a Reynolds number of 2.4×10^6 with the empirical curves. The agreement of the measurements with, in one case the turbulent curve and in the other case the laminar curve, is very good, so that there is no doubt that two different types of separation have occurred. It would at this point be justified to assume that the type of boundary layer existing at separation was improperly identified. However, perhaps an unsteady type of

laminar separation occurs, which is very similar to the turbulent separation. At present the analysis serves to point out that a paradox exists in the "laminar" separation bubble data.

Supersonic Separation

The extension of the subsonic separation results to supersonic flow is made by the approximate method developed by Reshotko and Tucker (ref. 19). The analysis is approximate in that the effects of friction are neglected. The moments-of-momentum equation is employed to calculate the change in the boundary-layer form factor through a discontinuity (shock wave). The form of the resulting solution suggests that the Mach-number ratio across a shock is a characteristic parameter for describing shock-induced separation. The analysis makes use of the Stewartson transformation, so that incompressible results are required to determine the compressible flow.

The Mach-number ratio across the shock wave according to Reshotko and Tucker's analysis is

$$\frac{M_2}{M_1} = \frac{f(H_2)}{f(H_1)} \quad (16)$$

where

$$f(H) = \frac{H^2}{(H^2 - 1)^{1/2}(H + 1)} e^{(1/H+1)} \quad (17)$$

In order to apply equation (16) together with the present separation criterion it is further assumed that δ^*/δ does not change across the

discontinuity. This assumption of constant δ^*/δ is compatible with the model of an abrupt discontinuity with no friction loss.

Reshotko and Tucker use a simple one-seventh power-law velocity profile to predict turbulent flat-plate shock-induced separation. For the one-seventh profile, $H_1 = 1.286$ and $\delta^*/\delta = 0.1250$. Reshotko and Tucker suggested a value of $H_2 = 2.2$ to obtain agreement with observed pressure rises. For $\delta^*/\delta = 0.1250$ the present analysis requires $H_2 = 2.143$ at separation. Figure 9(a) shows the results of the present criterion and that suggested by Reshotko and Tucker compared with measured pressure rises at shock-induced separation.

The present empirical velocity profile is in agreement with experimental results that neither H nor δ^*/δ are constants for the flat-plate turbulent boundary layer. Thus, no one curve based on a particular power profile would be expected to fit all the data of figure 9(a). A more realistic idea of the pressure rises expected at separation is obtained by considering the possible range for the values of H and δ^*/δ expected for the flat-plate turbulent boundary layer. Smith and Walker, (ref. 20) recently published measurements of an extensive survey of flat-plate turbulent boundary layers, ranging in Reynolds number (based on distance to leading edge of the plate) from 10^6 to 4×10^7 . The variation of δ^*/δ and H was from $\delta^*/\delta = 0.132$ and $H = 1.37$ for the low range of Reynolds numbers to $\delta^*/\delta = 0.0996$ and $H = 1.28$ for the highest Reynolds number. This gives Mach-number ratios for separation (eq. (16)) of 0.820 at the low Reynolds number and 0.757 at

the high Reynolds number. These two Mach-number ratios were used to determine the two dashed curves in figure 9(b). Although not necessarily the extreme limits on δ^*/δ , the two curves of figure 9(b) are adequate to bracket the data.

For laminar flow the analysis of Reshotko and Tucker is valid. However, the model of a sharp discontinuity can have only limited application to the laminar shock-separation problem. For laminar flow the shock wave does not penetrate the complete layer, thus a sharp discontinuity is not likely. For very large Reynolds numbers the model of the shock wave may be applicable, so the case of a Blasius value of H and δ^*/δ was computed. This case in which the Mach number ratio is 0.9845 is plotted in figure 10. The more realistic calculation for shock-induced laminar separation of Gadd (ref. 21) is also shown for comparison.

CONCLUSIONS

By using an assumed empirical equation for the mean velocity distribution as a guide, it has been possible to suggest a consistent model for intermittent turbulent separation. The basic hypothesis is that the outer region of the boundary layer has completely taken over the entire layer at separation. The outer region is already known to be equivalent to a laminar boundary layer with a larger viscosity, thus the intermittent turbulent separation profile is similar to a laminar boundary layer.

The model and thus the connection between turbulent separation and laminar boundary-layer flow is independent of the particular velocity-profile equation presented herein. The present analysis has also retained the empirical relation for laminar separation as a matter of convenience.

The value of the form factor at separation is given as a function of the ratio of displacement thickness to boundary-layer thickness. This relation falls short of allowing predictions of separation from free-stream conditions. It does, however, give some definite variation of form factor with Pohlhausen pressure gradient parameter at separation. The present results change the problem of predicting turbulent separation to a new one of determining the equivalence between flow conditions at the point of separation and known laminar boundary-layer solutions.

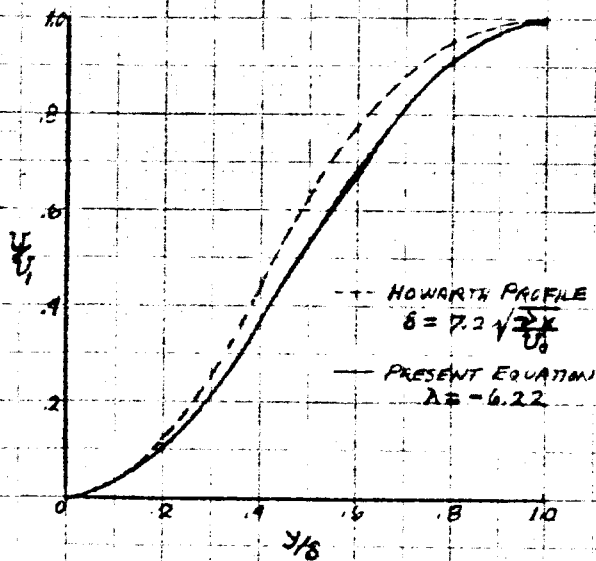
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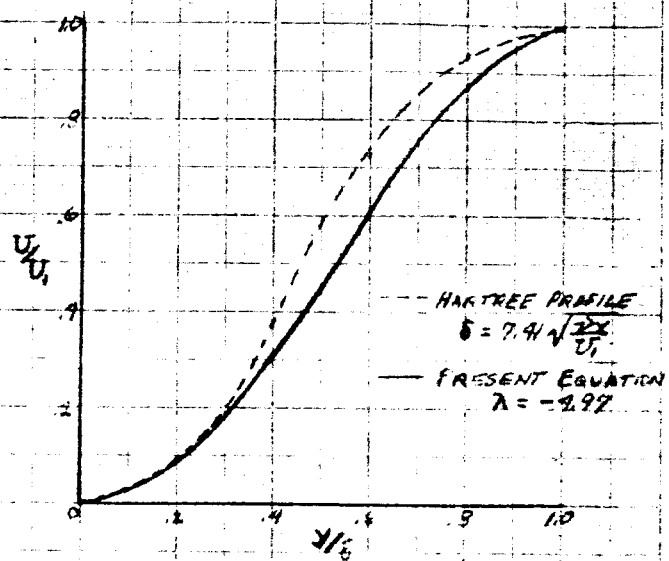
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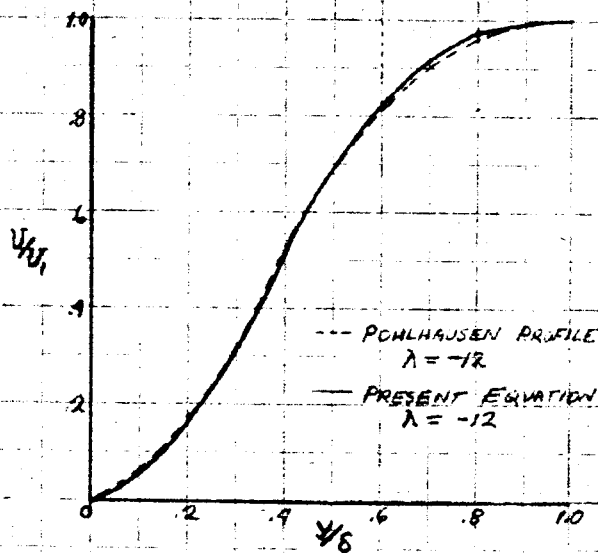
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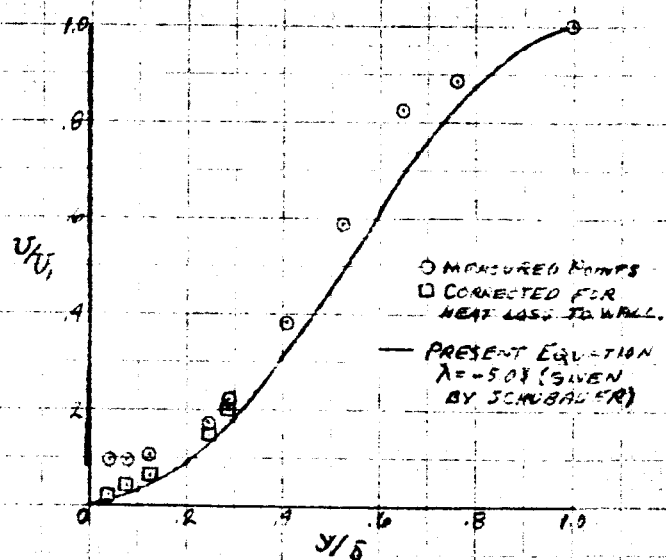
a) HOWARTH, LINEAR PRESSURE GRADIENT.



d) HARTREE, FALKNER-SKAN FLOW.



c) POHLHAUSEN SEPARATION PROFILE



e) SCHUBERGER, SEPARATION PROFILE ON
AN ELLIPTIC CYLINDER.

FIGURE 1 - COMPARISON OF PRESENT LAMINAR SEPARATION PROFILE EQUATION
WITH VARIOUS KNOWN SEPARATION PROFILES.

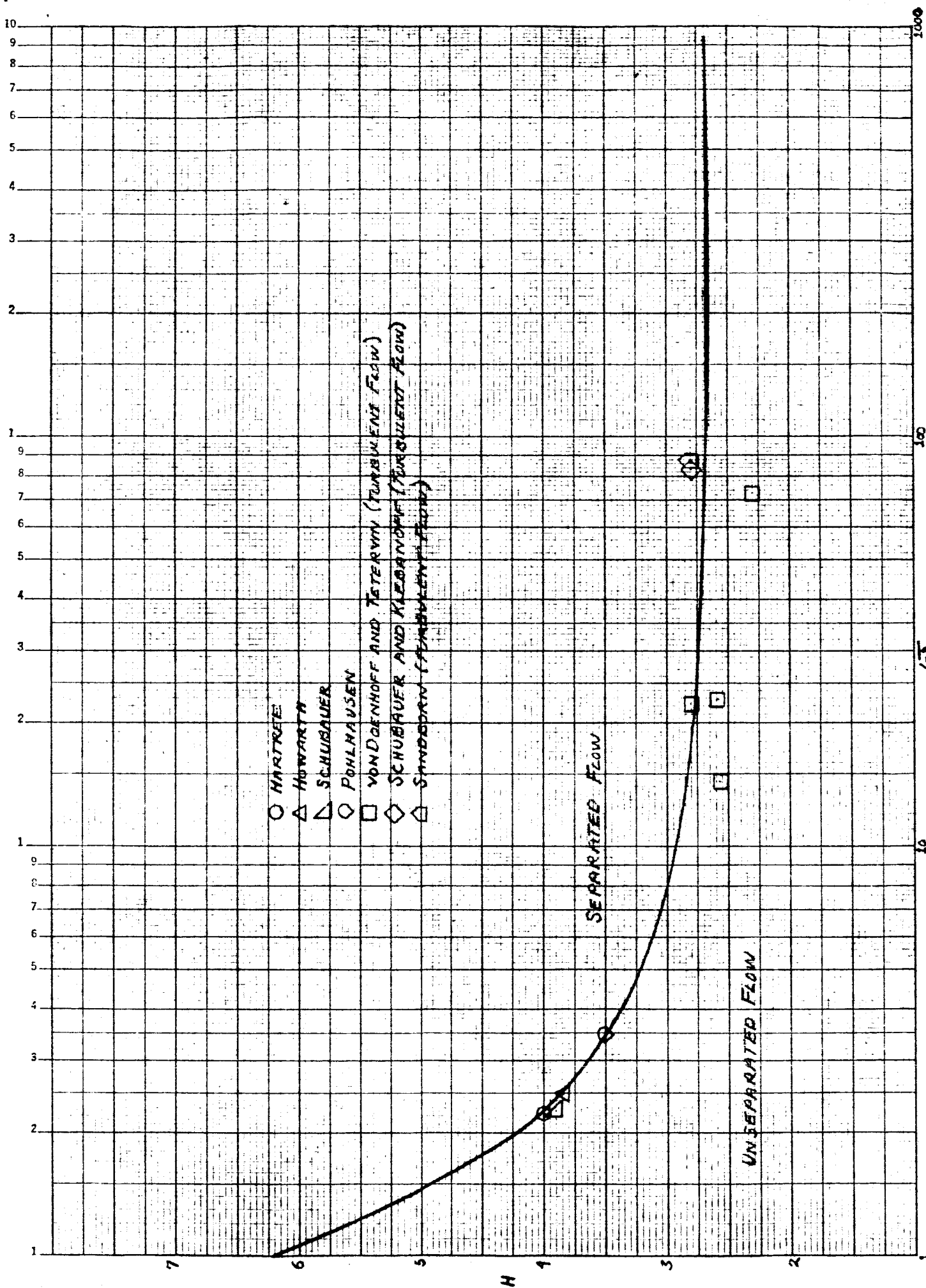


FIGURE 2. LAMINAR SEPARATION CRITERION.

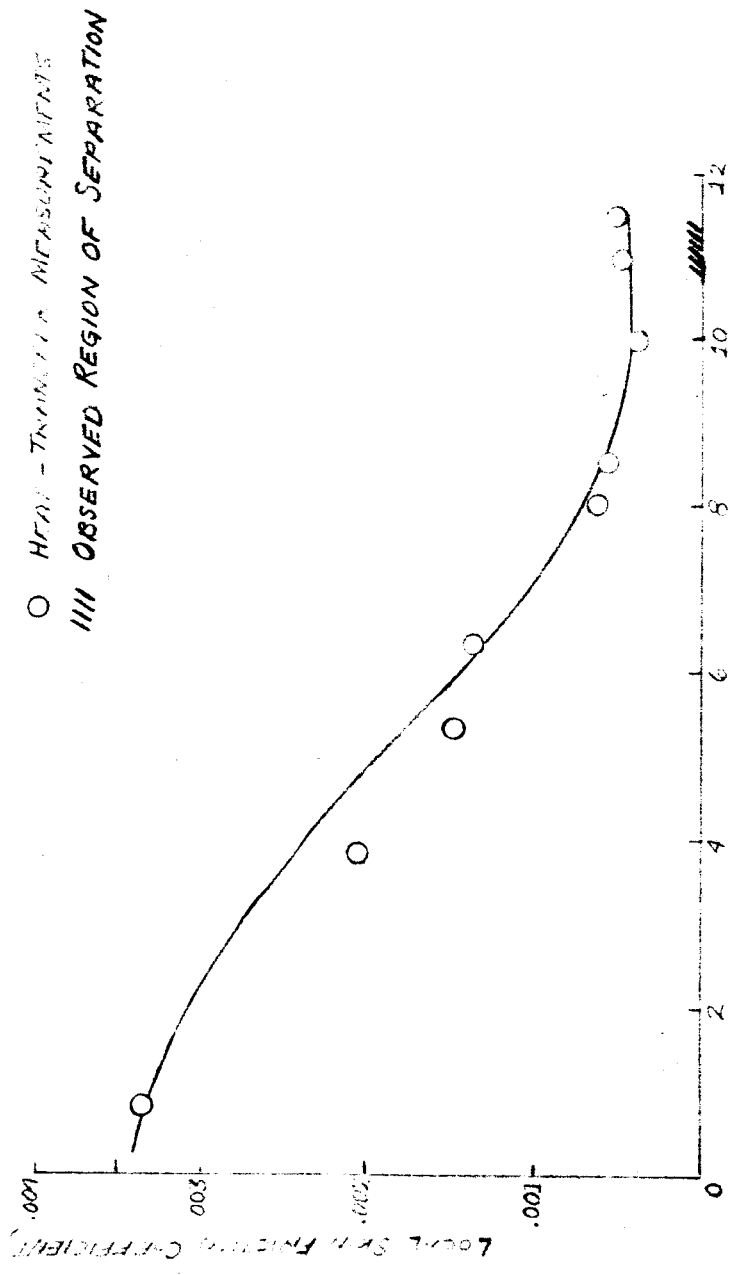


FIGURE 2. VARIATION OF LOCAL SPIN-FRICTION COEFFICIENT WITH DISTANCE FROM START OF TEST WHEEL, ILLUSTRATING REGION OF SEPARATION (REF. 13)

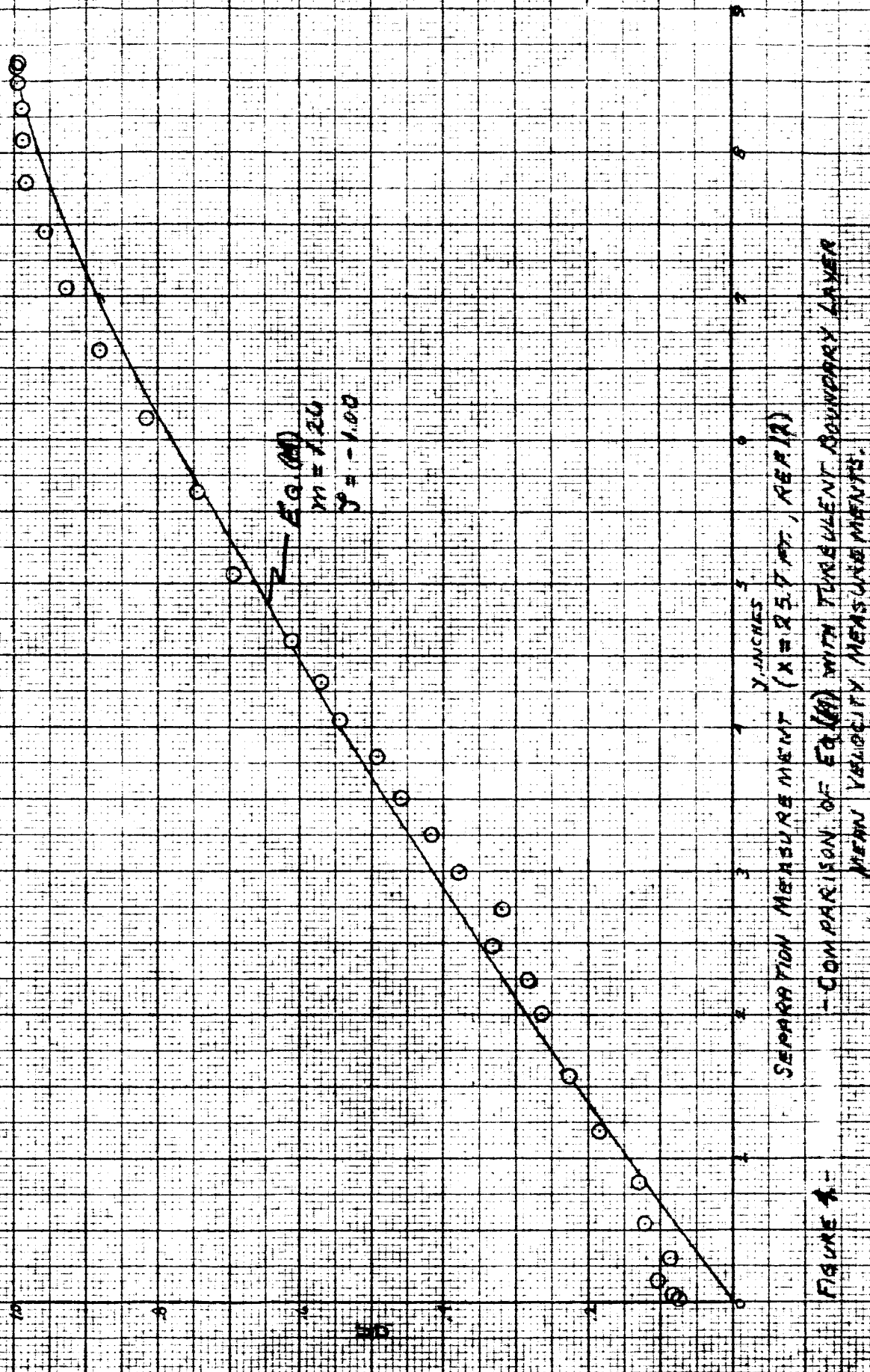


FIGURE 1-

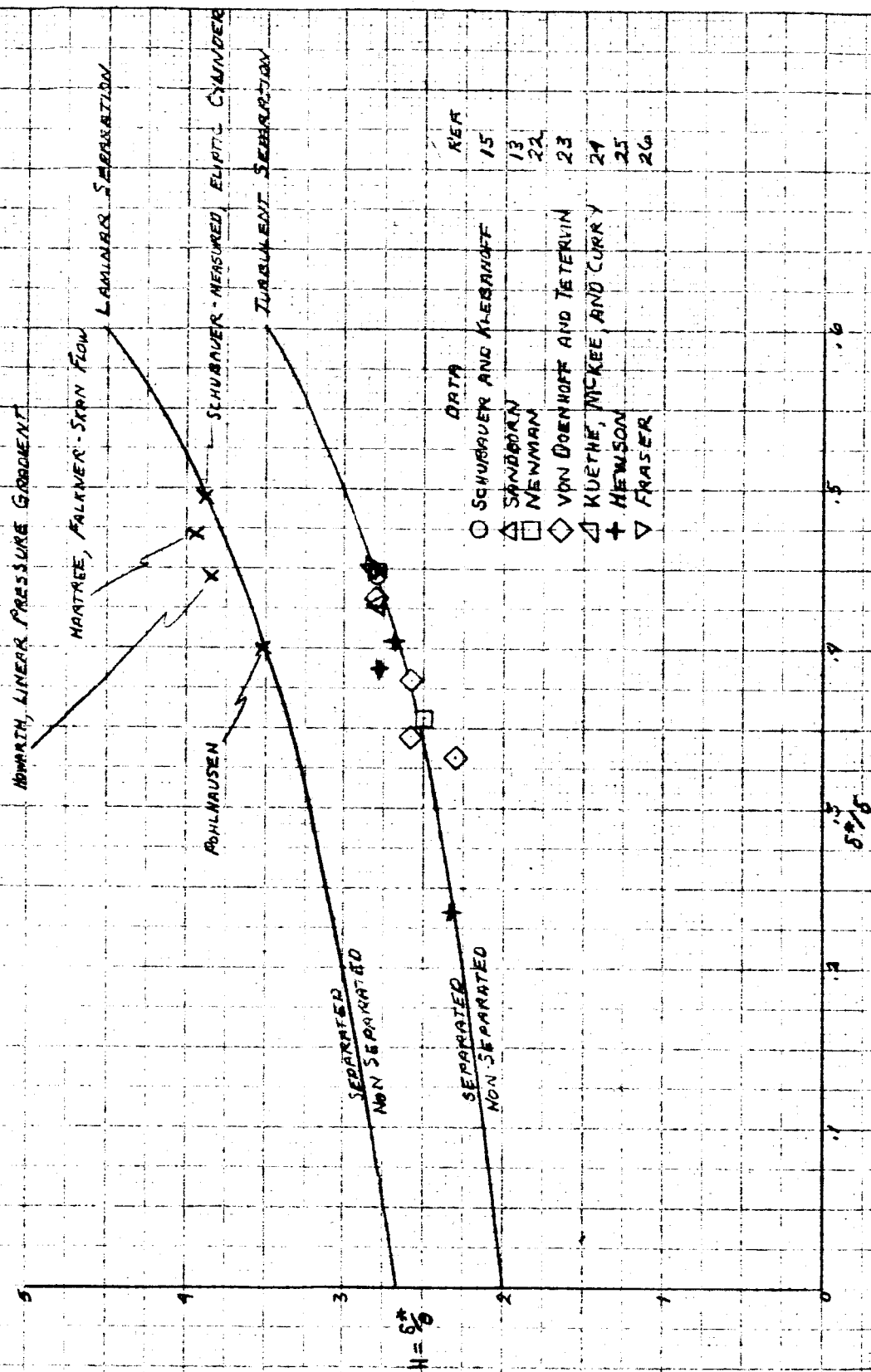


FIGURE 5.

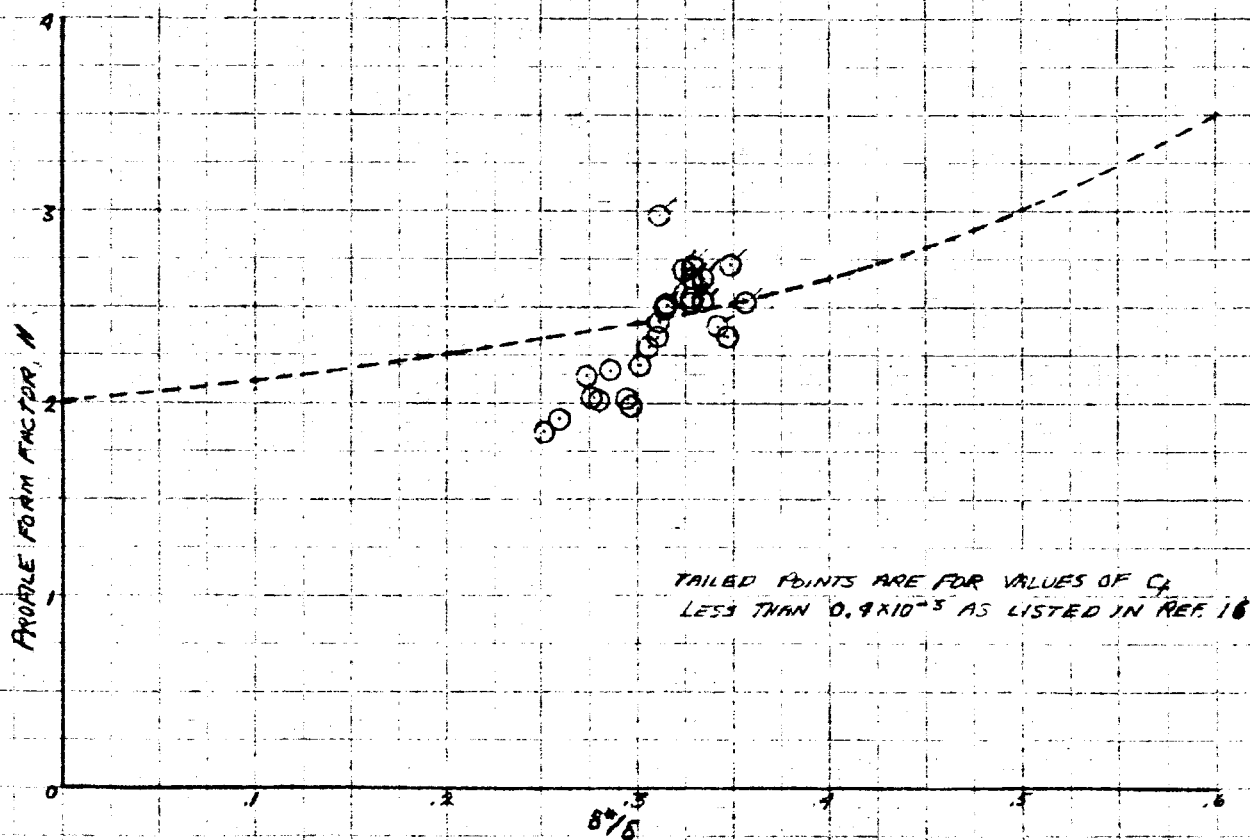


FIGURE 6 - COMPARISON OF MEASUREMENTS OF ROBERTSON AND HILL, REF. 18, WITH THE TURBULENT SEPARATION CRITERION.

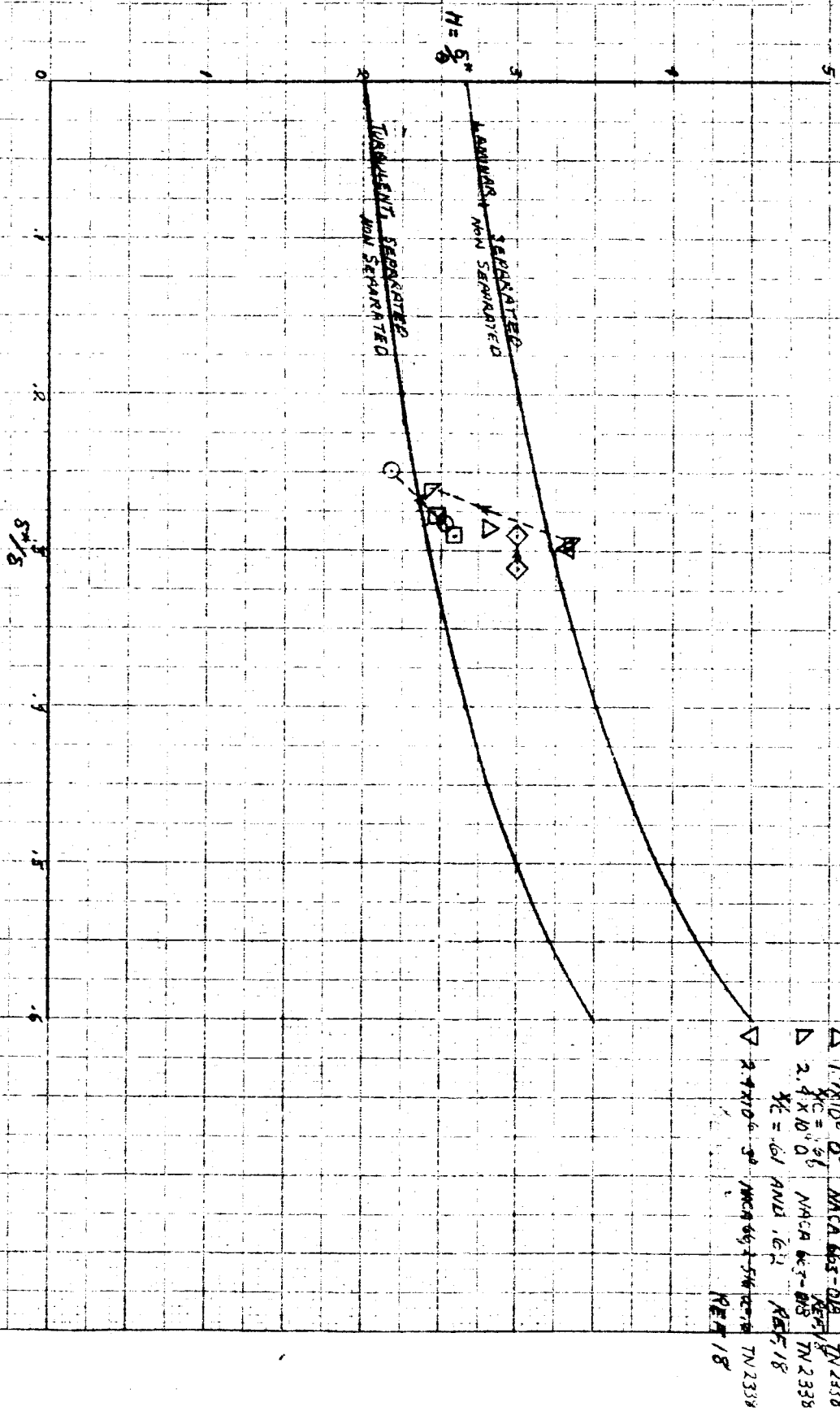


FIGURE 7 - COMPARISON OF "LAMINAR" SEPARATION BUBBLE DATA WITH THE LAMINAR AND TURBULENT SEPARATION CRITERIA.

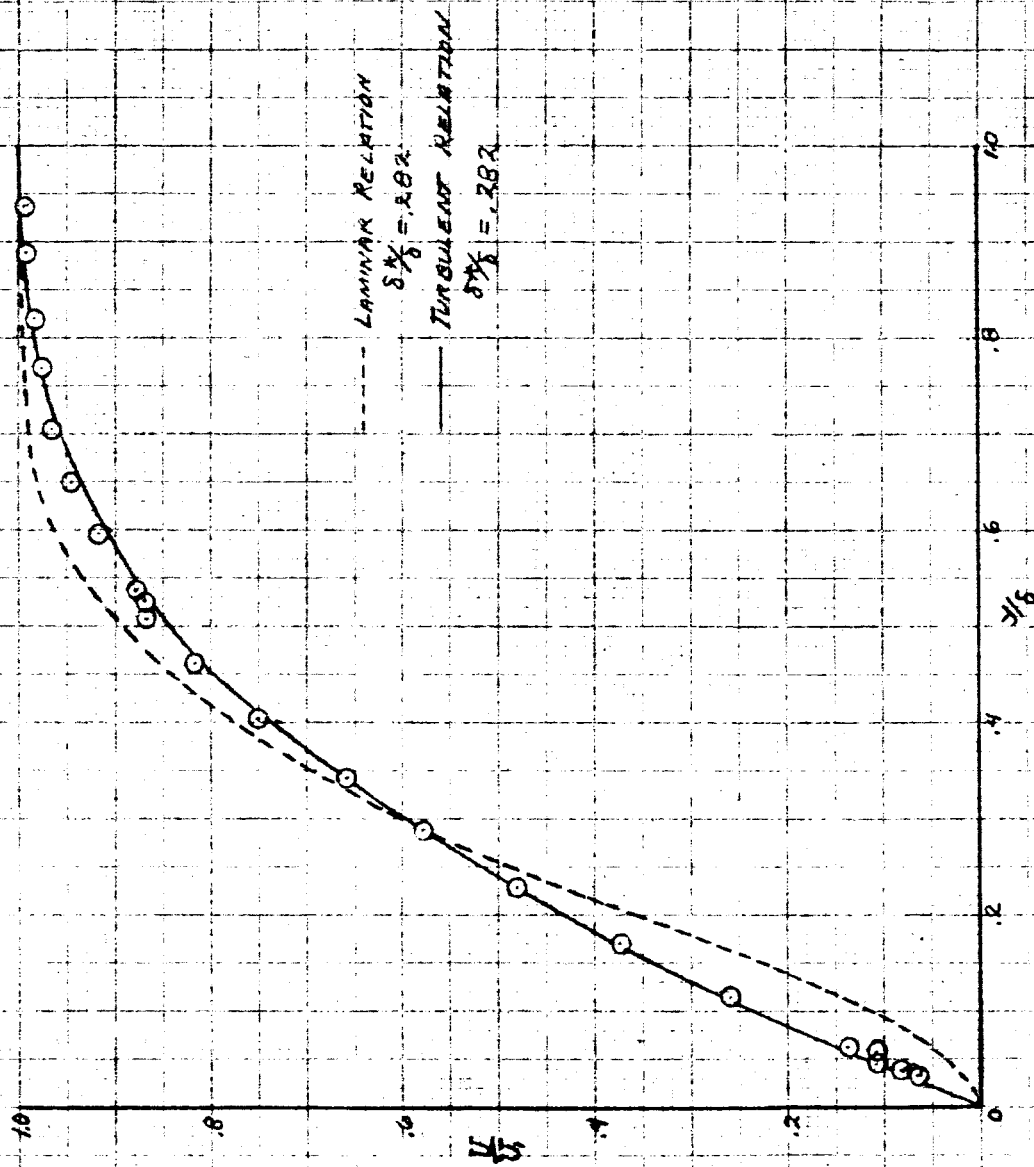


Fig. 8.9

"LAMINAR" SEPARATION PROFILE, $Re = 2 \times 10^5$, $NACA 663-018$ AIRFOIL, $Re = 2 \times 10^5$, $\alpha = 2^\circ$, $H = 2.58$, REF. 17

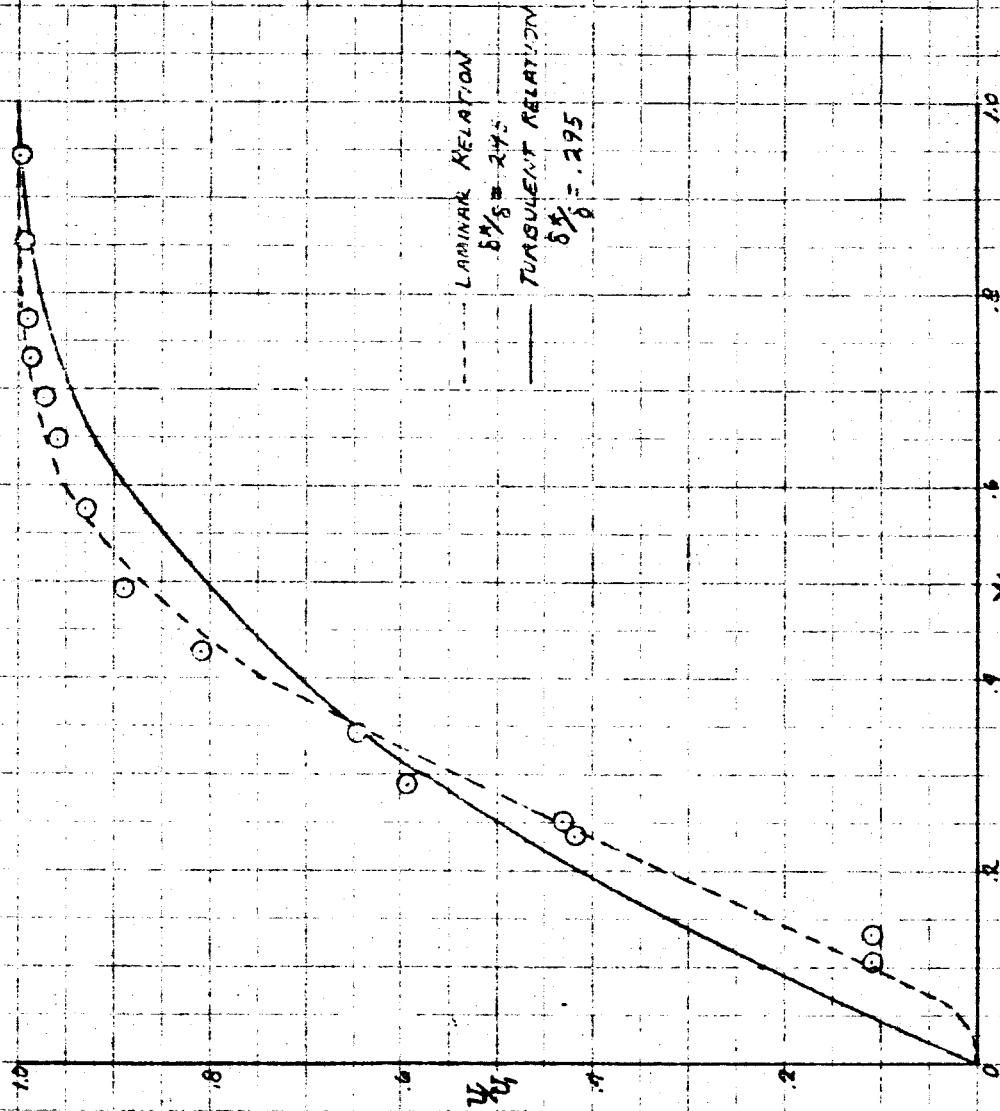


Fig. 8.61 LAMINAR SEPARATION PROFILE, $x_c = .01$, NACA 663-018 AIRFOIL, $Re = 2.4 \times 10^6$, $\alpha = 0^\circ$, $M = 3.30$, REF. 18.

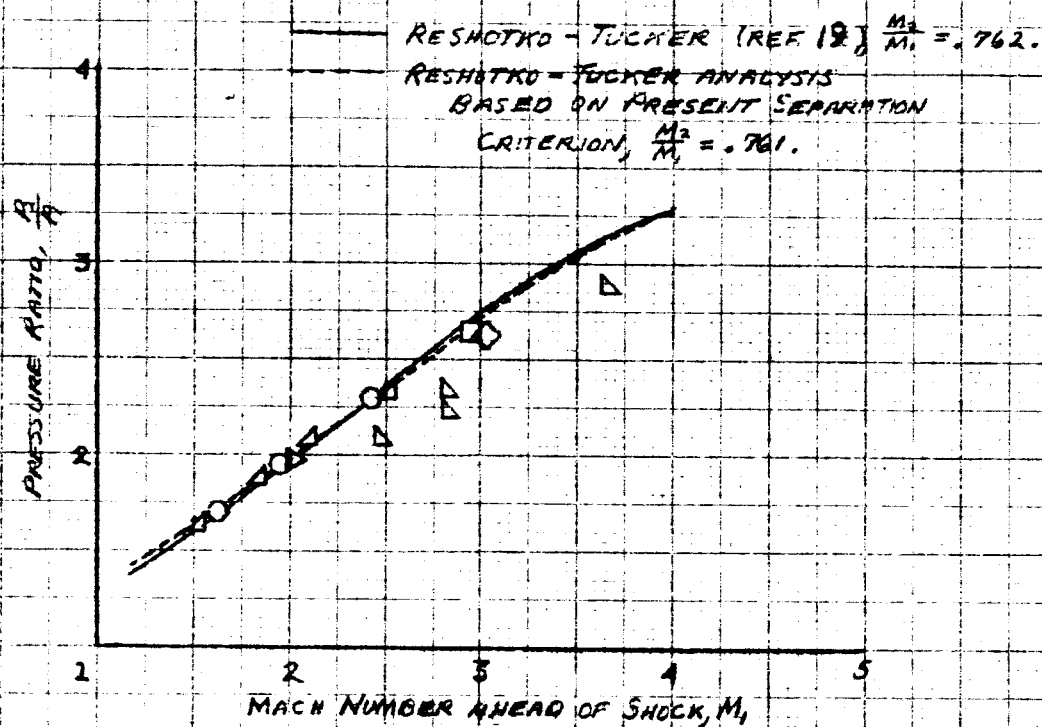


FIGURE 9. - PRESSURE RATIO FOR SHOCK-INDUCED, TURBULENT, BOUNDARY-LAYER SEPARATION FOR A ZERO PRESSURE GRADIENT.
a) COMPARISON OF PRESENT CRITERION FOR SEPARATION WITH MEASURED DATA. DATA POINTS IDENTIFIED IN REF. 18.

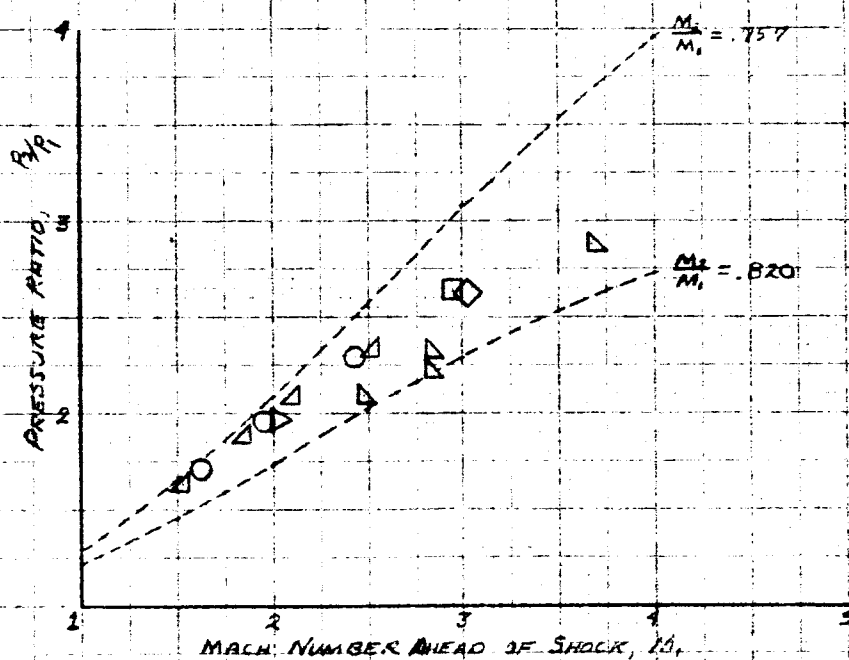


FIG. 9. - (CONT.)

b) BASED ON RANGE OF POSSIBLE VALUES OF
 8% OBSERVED FOR FLAT PLATE INCOMPRESSIBLE
 TURBULENT BOUNDARY LAYERS.

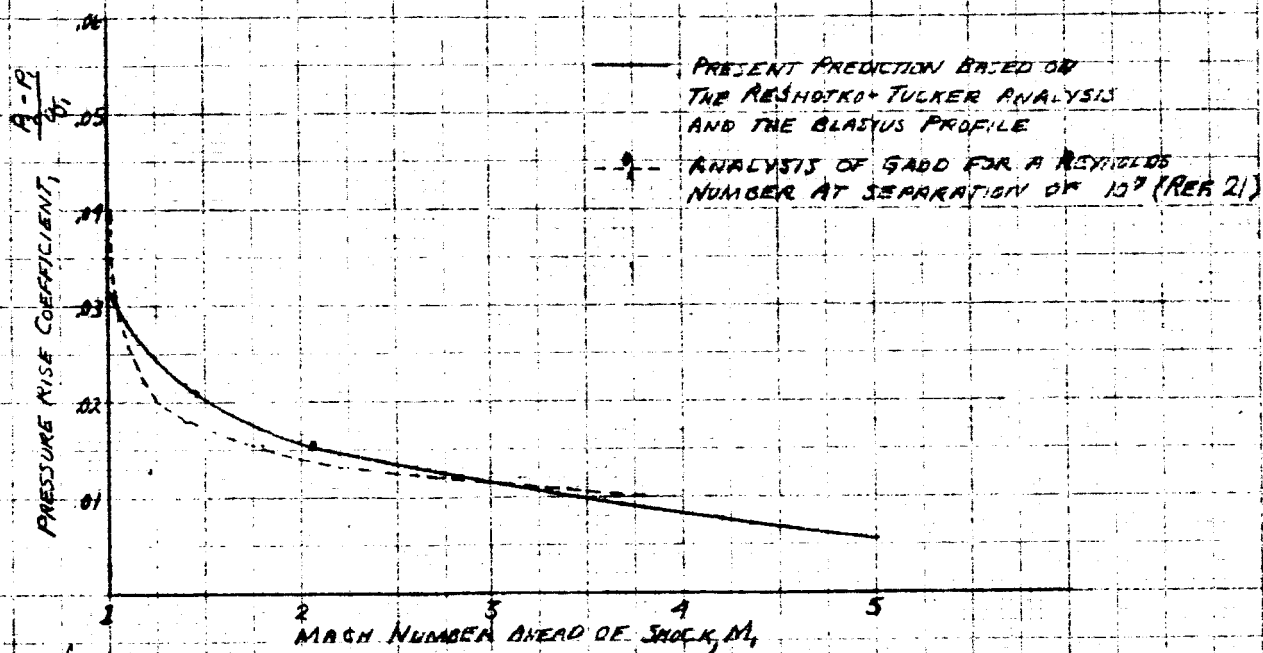


FIGURE 10. - PRESSURE RISE COEFFICIENT FOR SHOCK INDUCED LAMINAR BOUNDARY LAYER SEPARATION FOR A ZERO PRESSURE GRADIENT FLOW.